AP® Calculus BC (Operational)
2004 Sample Student Responses

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CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$\int_0^{30} \left[82+4\sin\left(\frac{t}{2}\right)\right]dt \approx 2474 \text{ cars}$$

2474 cars pass through over the 30-minute period.

Work for problem 1(b)

$$F(t) = 82+4\sin\left(\frac{t}{2}\right)$$

$$F'(t) = 4\cdot\frac{1}{2}\cdot\cos\left(\frac{t}{2}\right)$$

$$= 2\cos\left(\frac{t}{2}\right)$$

$$F'(7) = 2\cos\left(\frac{7}{2}\right) = -1.8729$$

Since the derivative of $F(t)$ is negative, the traffic flow is decreasing.

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Continue problem 1 on page 5.

A_2

Work for problem 1(c)

average value theo: $\dfrac{\int_a^b f(x)\,dx}{b-a}$

$$\frac{\int_{10}^{15}[82+4\sin(\frac{t}{2})]\,dt}{15-10}=\frac{409.4962}{5}=81.8992\approx 82 \text{ cars}/\text{min}$$

Work for problem 1(d)

$$\frac{\int_{10}^{15}F'(t)\,dt}{15-10}$$

$$=\frac{\int_{10}^{15}[2\cos(\frac{t}{2})]\,dt}{5}$$

$$=\frac{7.587697}{5}$$

$$=1.5175$$

$$\approx 2 \text{ cars}/\text{min}^2$$

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GO ON TO THE NEXT PAGE.

D1

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$\int_0^{30} \left[82+4\sin\left(\frac{t}{2}\right)\right]dt$$

$\boxed{2474 \text{ cars}}$

Work for problem 1(b)

$F(7) = 82 + 4\sin(7/2)$

$F(7) = 83.403$

Traffic flow is increasing because F(7) is positive.

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Continue problem 1 on page 5.

D2

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Work for problem 1(c)

$$\frac{1}{5}\int_{10}^{15}\left[82+4\sin\left(\frac{t}{2}\right)\right]dt =$$

81.899 cars

Work for problem 1(d)

$$\frac{F(15)-F(10)}{15-10} = \frac{85.752-78.164}{5}$$

1.518 cars per minute

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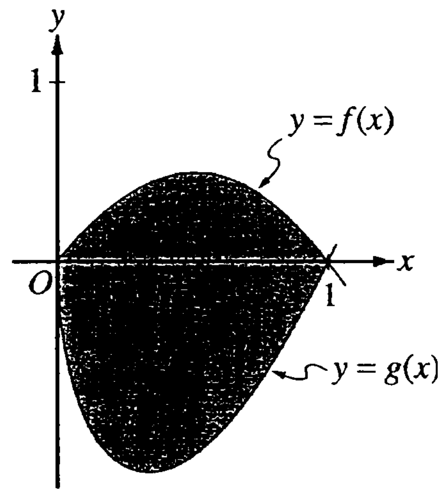

Work for problem 2(a)

$$\int_0^1 \left(2x(1-x)\right) - \left(3(x-1)\sqrt{x}\right) dx = 1.133$$

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Continue problem 2 on page 7.

Work for problem 2(b)

$$\pi\int_0^1 \left(2-3(x-1)\sqrt{x}\right)^2-\left(2-2x(1-x)\right)^2\,dx =$$

$$16.179$$

Work for problem 2(c)

$h(x)=kx(1-x) \quad 0\le x\le 1$

$$\int_0^1 \left[\left(kx(1-x)\right)-\left(3(x-1)\sqrt{x}\right)\right]^2 dx = 15$$

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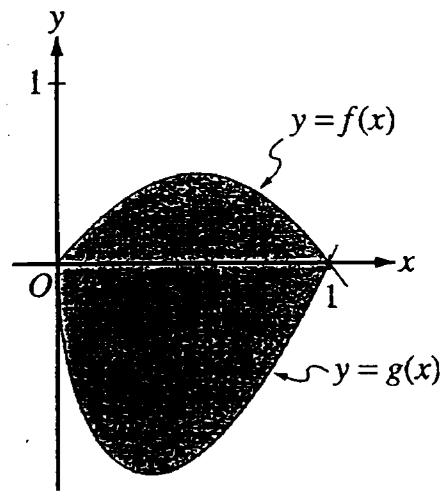

$0 \le x \le 1$

$f(x) = 2x(1-x)$
$g(x) = 3(x-1)\sqrt{x}$

Work for problem 2(a)

Area under $f(x) = \int_0^1 2x(1-x)\,dx \approx .\overline{333} = \frac{1}{3}$

Area under $g(x) = \int_0^1 3(x-1)\sqrt{x}\,dx \approx -.8$

make Area under $g(x)$ positive for total area

$|-.8| = .8$

Area enclosed by $f(x)$ & $g(x) = \frac{1}{3} + .8$

Area " " $= 17/15 \approx 1.1\overline{333}$ ✓

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Continue problem 2 on page 7.

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Work for problem 2(b)

Volume of solid

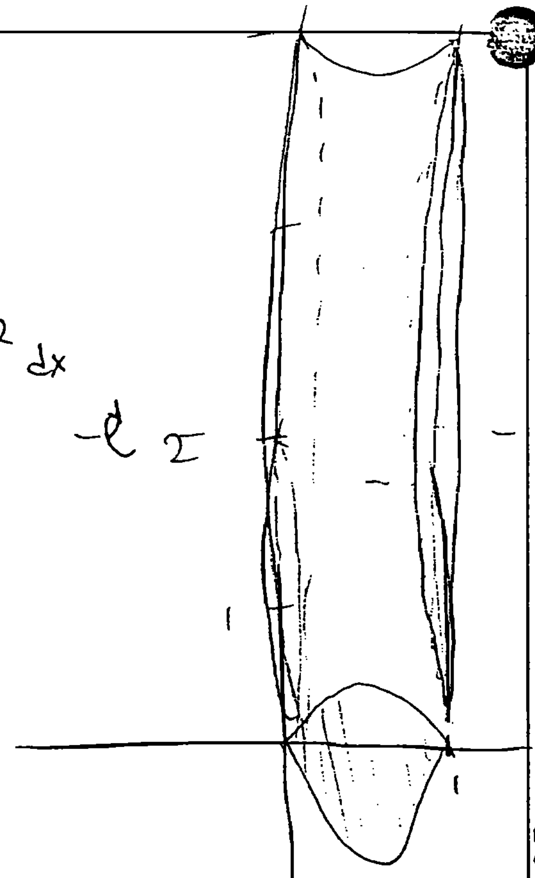
$$\pi\int_0^1 R^2(x) - r^2(x)\,dx$$

(outside function)
$R(x)$
$2 - g(x)$
$R(x) = 2 - 3(x-1)\sqrt{x}$

$$\pi\int_0^1 (2-3(x-1)\sqrt{x})^2 - (2-2x(1-x))^2\,dx$$

$$= \frac{103\pi}{20} \approx 16.179$$

$-\ell\ 2$

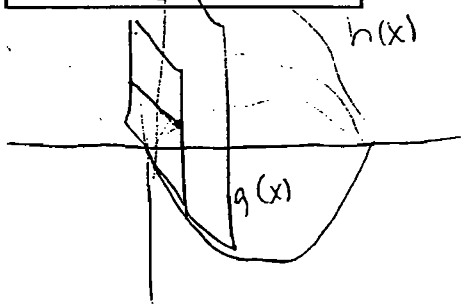


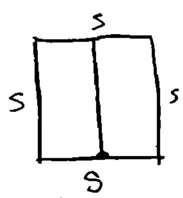
(inside function)
$r(x)$
$2 - f(x)$
$r(x) = 2 - 2x(1-x)$

Work for problem 2(c)

$0 \le x \le 1$
$h(x) = kx(1-x)$

$k > 0$





$A_{\square} = s^2$

$s = h(x) - g(x)$

$$A_{\text{all}\,\square} = \pi\int_0^1 kx(1-x) - 3(x-1)\sqrt{x}\,dx$$

$$16 = \pi\int_0^1 kx(1-x) - 3(x-1)\sqrt{x}\,dx$$ use to find a 'k' value.

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B1

Work for problem 3(a)

$\frac{dx}{dt} = 3+\cos(t^2)$

$\int dx = \int(3+\cos(t^2))dt$

$x = \int(3+\cos(t^2))dt$

(a) $\int_2^4 (3+\cos(t^2)) = 6.1330$

$x(4) = x(2) + 6.1330 = 1 + 6.1330 = 7.1330$

$\boxed{x(4) = 7.1330}$

Work for problem 3(b)

$\left.\frac{dx}{dt}\right|_{t=2} = 3 + \cos(2^2) = 2.3464$

$\text{slope} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-7}{2.3464} = -2.9833$

$8 = (-2.9833)(1) + b$

$b = 10.9833$

$\boxed{y = -2.9833 + 10.9833}$

Do not write beyond this border.

B2

Work for problem 3(c)

$\left.\frac{dx}{dt}\right|_{t=2} = 3+\cos(2^2) = 2.3464$

$\left.\frac{dy}{dt}\right|_{t=2} = -7$

speed $= |v| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(2.3464)^2 + (-7)^2} = \boxed{7.3828}$

Work for problem 3(d)

$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2t+1 = \frac{\frac{dy}{dt}}{3+\cos(t^2)}$

$\frac{dy}{dt} = (2t+1)(3+\cos(t^2))$

$\frac{d^2x}{dt^2} = -2t\sin(t^2)\Big|_{t=4} = -2(4)\sin(4^2) = 2.3032$

$\frac{d^2y}{dt^2} = (2t+1)(-2t\sin t^2) + 2(3+\cos(t^2))\Big|_{t=4}$

$= (2(4)+1)(-2(4)\sin(4^2)) + 2(3+\cos(4^2)) = 24.8137$

$\boxed{a = (2.3032, 24.8137)}$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Do not write beyond this border.

C_1

Work for problem 3(a)

$x(4) = 1 + \int_2^4 3 + \cos(t^2)$

$x(4) = 7.133$

Work for problem 3(b)

at $t = 2$ $\frac{dx}{dt} = 3 + \cos(2^2)$

$\frac{dy}{dx} = \frac{7}{2.3464}$

$\frac{dy}{dx} = 2.9833$

$(y - 8) = 2.9833(x - 1)$

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Work for problem 3(c)

$S = \sqrt{dy^2 + dx^2}$ at $t=2$

$S = 7.3827$

Work for problem 3(d)

$\frac{dy}{dx} = 2t + .1$

$dx = 3 + \cos(t^2)\,dt$

$dy = (2t+1)(3+\cos(t^2))\,dt$

$\frac{dy}{dt} = (2t+1)(3+\cos(t^2))$

at $t=4$ $\frac{d^2y}{dt^2} = 24.814$

at $t=4$ $\frac{d^2x}{dt^2} = 2.3032$

$A = \sqrt{2.3032^2 + 24.814^2}$ $A = 24.9207$

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END OF PART A OF SECTION II

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2004 Sample Student Responses

NO CALCULATOR ALLOWED

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$x^2 + 4y^2 = 7 + 3xy$

$2x + \underline{4 \cdot 2y \cdot y'} = 0 + \underline{3xy'} + 3y$

$8y \cdot y' - 3x \cdot y' = 3y - 2x$

$y' \cancel{(8y - 3x)} = \dfrac{3y - 2x}{8y - 3x}$

$y' = \dfrac{3y - 2x}{8y - 3x}$ ✓

Do not write beyond this border.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$x^2 + 4y^2 = 7 + 3xy \qquad x = 3$

$9 + 4y^2 = 7 + 9y \qquad 4y^2 - 9y + 2 = 0 \quad y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{9 \pm \sqrt{81 - 4\cdot 2\cdot 4}}{2\cdot 4} = \frac{9 \pm \sqrt{81 - 32}}{8} = \frac{9 \pm \sqrt{49}}{8} = \frac{9 \pm 7}{8} = \frac{16}{8}, \frac{2}{8}$

$y = 2, \frac{1}{4}$

$\frac{3y - 2x}{8y - 3x} = 0 \qquad 3y - 2x = 0 \qquad 3y = 2x$

$3y = 2\cdot 3$

$y = 2$ ✓

$P(3,2)$

$\frac{3\cdot 2 - 2\cdot 3}{8\cdot 2 - 3\cdot 3} = \frac{0}{7}$ ✓ $= 0$

$16 - 9$

Work for problem 4(c)

$\frac{3y - 2x}{8y - 3x} = y'$

$8y\cdot y' - 3x\cdot y' = 3y - 2x$

$\underline{8y\cdot y''} + 8y'\cdot y' - \underline{3x\cdot y''} - 3y' = 3y' - 2$

$y''(8y - 3x) = 3y' - 2 - 8y'^2 + 3y' =$

$y'' = \frac{3y' - 2 - 8y'^2 + 3y'}{8y - 3x}$

~~$y''(2) \rightarrow y'(2) =$~~

~~$x = 3$~~

~~$y(2) = 2$~~

$\boxed{y' = 0 \quad x = 3 \quad y = 2}$

$y'' = \frac{0 - 2 - 0 + 0}{8\cdot 2 - 3\cdot 3} = \frac{-2}{16 - 9} = -\frac{2}{7} < 0$

$y'' < 0$

concave down

maximum

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$x^2 + 4y^2 = 7 + 3xy$

$\frac{dy}{dx} = \frac{3y-2x}{8y-3x}$ show

$2x \cdot \cancel{\frac{dx}{dx}} + 8y\frac{dy}{dx} = 0 + 3\left(x \cdot \frac{dy}{dx} + y\right)$

$2x + 8y\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y$

$8y\frac{dy}{dx} - 3x\frac{dy}{dx} = 3y - 2x$

$\frac{dy}{dx}(8y - 3x) = 3y - 2x$

$\frac{dy}{dx} = \boxed{\frac{3y-2x}{8y-3x}}$ ✓

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Continue problem 4 on page 11.

Work for problem 4(b)

$P(3, \underline{2})$

$m = 0$

$$\frac{dy}{dx} = m = \frac{3y - 2x}{8y - 3x}$$

$$\frac{3y - 2\cdot 3}{8y - 3\cdot 3} = 0$$

$3y - 6 = 0$

$$\frac{3y - 6}{8y - 9} = 0 \qquad \boxed{y = 2}$$

Work for problem 4(c)

$P(3,2)$ $\qquad \dfrac{dy}{dx} = \dfrac{3y-2x}{8y-3x}$

$$\frac{d^2y}{dx} = \frac{(8y-3x)\left(3\frac{dy}{dx} - 2\right) - (3y-2x)\left(8\frac{dy}{dx} - 3\right)}{(8y-3x)^2}$$

16 −9 0

$$\frac{d^2y}{dx} = \boxed{\frac{(8\cdot 2 - 3\cdot 3)(-2) - (3\cdot 2 - 2\cdot 3)(-3)}{(8\cdot 2 - 3\cdot 3)^2}}$$

$$\frac{d^2y}{dx} = \frac{-14 - 0}{49} = -\frac{14}{9}$$

$\dfrac{3y - 2x}{8y - 3x}$ ~~[illegible]~~

The curve has a local maximum because at pt. P(3,2), the 2^{nd} derivative is negative. This makes the curve concave down.

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AP® Calculus BC (Operational)
2004 Sample Student Responses

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NO CALCULATOR ALLOWED

A1

Work for problem 5(a)

$$\frac{dP}{dt} = \frac{P}{60}(12 - P)$$

If $P(0) = 3$, $\lim_{t\to\infty} P(t) =$ 12

If $P(0) = 20$, $\lim_{t\to\infty} P(t) =$ 12

$$P(t) = \frac{12}{1 + Ce^{-t/5}}$$

$$P(0) = \frac{12}{1+C} = 3$$

$c =$

$$P(t) = \frac{12}{1+3e^{-t/5}}$$

$$P(0) = \frac{12}{1+C} = 20$$

$c = -.4$

$$P(t) = \frac{12}{1-.4e^{-t/5}}$$

Work for problem 5(b)

$\frac{1}{2}$ max population is when pop. grows fastest.

and $\lim_{t\to\infty} P(t) = 12$ ∴

$$\frac{12}{2} = 6$$

Continue problem 5 on page 13.

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NO CALCULATOR ALLOWED

A2

Do not write beyond this border.

Work for problem 5(c)

$$\int \frac{dy}{y} = \int \left(\frac{1}{5} - \frac{t}{60}\right) dt$$

$$\ln y = \frac{t}{5} - \frac{t^2}{120} + c$$

$$y = Ce^{\left(\frac{t}{5} - \frac{t^2}{120}\right)}$$

$$y(0) = Ce^0 = 3$$

$$c = 3$$

$$y(t) = 3e^{\left(\frac{t}{5} - \frac{t^2}{120}\right)}$$

Work for problem 5(d)

$$\lim_{t \to \infty} 3e^{\left(\frac{t}{5} - \frac{t^2}{120}\right)} = 0$$

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GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

C 1

Work for problem 5(a)

If $P(0) = 3$, $\lim_{t\to\infty} P(t) =$ 12

If $P(0) = 20$, $\lim_{t\to\infty} P(t) =$ 12

The population will reach an equilibrium when $1 - \frac{P}{12} = 0$, i.e., when $P = 12$

Work for problem 5(b)

$\frac{P}{5}\left(1 - \frac{P}{12}\right) = \frac{P}{5} - \frac{P^2}{60}$ reaches its maximum when

$\frac{1}{5} - \frac{P}{30} = 0 \qquad P = 6$

$\therefore \underline{\underline{P = 6}}$

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Continue problem 5 on page 13.

C_2

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Work for problem 5(c)

$\frac{5}{Y} dY = (1 + \frac{t}{12}) dt$

$\int \frac{5}{Y} dY = \int (1 + \frac{t}{12}) dt$

$(Y \geq 0)$ $5 \ln Y = t + \frac{t^2}{6} + c$

$\ln Y = \frac{t}{5} + \frac{t^2}{30} + c$

$Y = Ce^{\frac{t}{5}} e^{\frac{t^2}{30}}$

$Y(0) = C = 3$

$\therefore$ $Y(t) = 3e^{\frac{t}{5}} \cdot e^{\frac{t^2}{30}}$ //

Work for problem 5(d)

$\lim_{t \to \infty} Y(t) = \infty$ //

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NO CALCULATOR ALLOWED

Work for problem 6(a)

$f(x) = \sin(5x + \frac{\pi}{4})$ $\quad f(0) = \frac{\sqrt{2}}{2}$

$f'(x) = \cos(5x + \frac{\pi}{4}) \cdot 5$ $\quad f'(0) = \frac{5}{2}\sqrt{2}$

$f''(x) = -5\sin(5x + \frac{\pi}{4}) \cdot 5$ $\quad f''(0) = -\frac{25}{2}\sqrt{2}$

$f'''(x) = -25\cos(5x + \frac{\pi}{4}) \cdot 5$ $\quad f'''(0) = -\frac{125}{2}\sqrt{2}$

$$\therefore P(x) = \frac{\sqrt{2}}{2} + \frac{\frac{5}{2}\sqrt{2}}{1!}x - \frac{\frac{25}{2}\sqrt{2}}{2!}x^2 - \frac{\frac{125}{2}\sqrt{2}}{3!}x^3$$

$$= \frac{\sqrt{2}}{2} + \frac{5}{2}\sqrt{2}x - \frac{25}{4}\sqrt{2}x^2 - \frac{125}{12}\sqrt{2}x^3$$

Work for problem 6(b)

$$\left|f^{(n)}(0)\right| = \frac{5^n}{2}\sqrt{2}$$

signs alternate every 2 terms

$\therefore \frac{22+1}{4} = 5 \cdots \boxed{3}$ - remainder.

$\therefore$ sign negative

$$\therefore \text{coefficient} = -\frac{\frac{5^{22}}{2}\sqrt{2}}{22!} = -\frac{5^{22}\sqrt{2}}{2 \cdot 22!}$$

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Continue problem 6 on page 15.

Work for problem 6(c)

$|f(\frac{1}{10}) - P(\frac{1}{10})| = \frac{f^4(c)}{4!}(\frac{1}{10})^4$ for $0 < c < \frac{1}{10}$

$f^4(c) = 125 \cdot \sin(5c + \frac{\pi}{4}) \cdot 5$ for $0 < c < \frac{1}{10}$

$\frac{\pi}{4} < 5c + \frac{\pi}{4} < \frac{1}{2} + \frac{\pi}{4} = \frac{2+\pi}{4} < \pi$

$\therefore \sin(5c + \frac{\pi}{4}) > 0$ and $\sin(5c + \frac{\pi}{4}) < 1$

$\therefore f^4(c) < 125 \cdot 5 = 625$

$\therefore \frac{f^4(c)}{4!}(\frac{1}{10})^4 < \frac{625}{24} \cdot \frac{1}{10000} = \frac{1}{24 \cdot 16} = \frac{1}{384} < \frac{1}{100}$

$\therefore |f(\frac{1}{10}) - P(\frac{1}{10})| < \frac{1}{100}$

Work for problem 6(d)

$G(x) = \int_0^x f(t)\,dt$ } fundamental theorem of calculus.

$\therefore G'(x) = f(x)$

$G''(x) = f'(x)$

Taylor polynomial for G about $x=0$ is an antiderivate of $P(x)$

$\therefore P(x) = \frac{\sqrt{2}}{2} + \frac{5}{2}\sqrt{2}x - \frac{25}{4}\sqrt{2}x^2 - \frac{125}{12}\sqrt{2}x^3$

let $k(x)$ be the ~~tay~~ taloy polynomial for G about $x=0$

$\therefore \boxed{k(x) = \frac{\sqrt{2}}{2}x + \frac{5}{4}\sqrt{2}x^2 - \frac{25}{12}\sqrt{2}x^3}$

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.**
- **MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.**

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Work for problem 6(a)

$f(x) = \sin(5x + \frac{\pi}{4})$

$f(0) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$f'(c) = 5\cos(5c + \frac{\pi}{4}) \quad f'(0) = \frac{5}{\sqrt{2}}$

$f''(c) = -25\sin(5c + \frac{\pi}{4}) \quad f''(0) = \frac{-25}{\sqrt{2}}$

$f'''(c) = -125\cos(5c + \frac{\pi}{4}) \quad f'''(0) = \frac{-125}{\sqrt{2}}$

$P(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!}$

$\boxed{P(x) = \frac{1}{\sqrt{2}} + \frac{5x}{\sqrt{2}} - \frac{25}{2\sqrt{2}}x^2 - \frac{125}{6\sqrt{2}}x^3}$

Work for problem 6(b)

$\cancel{x^n = 5^n}$

coefficient of $x^n = \frac{5^n}{n!\sqrt{2}}$

coefficient of $x^{22} = \boxed{\frac{5^{22}}{22!\sqrt{2}}}$

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Continue problem 6 on page 15.

MM2

6 6 6 6 6 6 6 6 6 6

NO CALCULATOR ALLOWED

Work for problem 6(c)

Lagrange error $\frac{f^{(n+1)}(z)(x-c)^{n+1}}{(n+1)!}$

$f^{iv}(z) = 625(\sin(5z+\frac{\pi}{4}))$

$|f(\frac{1}{10}) - P(\frac{1}{10})| \le \cancel{\frac{f^{iv}(z)(\frac{1}{10})}{4!}} \le \frac{f^{iv}(z)(x)^4}{4!} < \frac{1}{100}$ $\quad c<z<x$, $0<z<\frac{1}{10}$

$\le \frac{f^{iv}(z)(\frac{1}{10})^4}{4!} < \frac{1}{100}$

$\le \frac{625(\sin(\frac{5}{10}+\frac{\pi}{4}))\frac{1}{10000}}{24} < \frac{1}{100}$

$\le \frac{\frac{625}{10000}\sin(\frac{2+\pi}{4})}{24} < \frac{1}{100}$

$\le \frac{625\sin(\frac{2+\pi}{4})}{240000} < \frac{1}{100}$

$|f(\frac{1}{10}) - P(\frac{1}{10})| \le \frac{\sin(\frac{2+\pi}{4})}{384} < \frac{1}{100}$

$\boxed{\sin(\frac{2+\pi}{4}) < 1 \quad \frac{1}{384} < \frac{1}{100}}$

Work for problem 6(d)

$G(x) = C + \frac{1}{\sqrt{2}}x + \frac{5x^2}{2\sqrt{2}} - \frac{25x^3}{6\sqrt{2}} \cancel{+ \frac{125x^4}{24\sqrt{2}}}$

$G(0) = 0 \quad C = 0$

$\cancel{G(x) = \frac{1}{\sqrt{2}}x + \frac{5x^2}{2\sqrt{2}} - \frac{25x^3}{6\sqrt{2}}}$

$\boxed{G(x) = \frac{1}{\sqrt{2}}x + \frac{5x^2}{2\sqrt{2}} - \frac{25x^3}{6\sqrt{2}}}$

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.**
- **MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.**

Do not write beyond this border.